HEAT TRANSFER FOR THE CYLINDRICAL FILM SENSOR OF A THERMAL ANEMOMETER

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The authors investigate heat transfer for the cylindrical film sensor of a thermal anemometer.

The cylindrical thermal anemometer sensor used in experimental liquid and gas mechanics to study three-dimensional flows of moving media is a cylinder of dielectric material with a metal film on its surface [1]. Heat is removed from the heated film element to the stream washing the sensor and to the substrate. To study the operation, one must know the form of the thermal field inside and outside the sensor. We shall consider a model, a finite cylinder of length $2l$ with a surface heat source of length 2c (Fig. 1). Then the heat-conduction equation for the cylindrical substrate for steady conditions has the form [2]

$$\frac{\partial^2}{\partial r^2}T(r, z, \varphi)+\frac{1}{r}\frac{\partial}{\partial r}T(r, z, \varphi)+\frac{\partial^2}{\partial r^2}T(r, z, \varphi)+\frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}T(r, z, \varphi)=0. \quad (1)$$

The thermal field in the boundary layer of a heated cylinder washed by a stream was considered in detail in [3]. The local heat transfer to the flow depends on the angle φ and up to Re = 5000 its value on the forward part of the cylinder is greater than on the rear.

Because of the large thermal conductance of the sensor film element the temperatures of its peripheral zones do not differ significantly. The temperature field of the cylindrical substrate under the surface heat source will be close to axisymmetric, and Eq. (1) takes the form:

$$\frac{\partial^2}{\partial r^2}T(r, z)+\frac{1}{r}\frac{\partial}{\partial r}T(r, z)+\frac{\partial^2}{\partial z^2}T(r, z)=0. \quad (2)$$

The boundary conditions are:

$$-\lambda\frac{\partial}{\partial r}T(R, z)+g-\alpha[T(R, z)-T_{\mathrm{m}}]=0, \quad (3)$$

$$T(0, z)\neq\infty, \quad (4)$$

$$\frac{\partial}{\partial z}T(r, 0)=0, \quad (5)$$

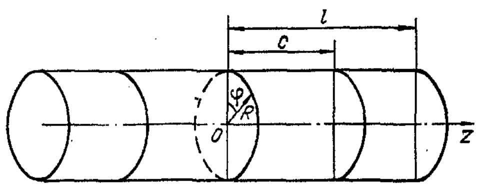

Fig. 1. Model of the cylindrical film sensor of the thermal anemometer.

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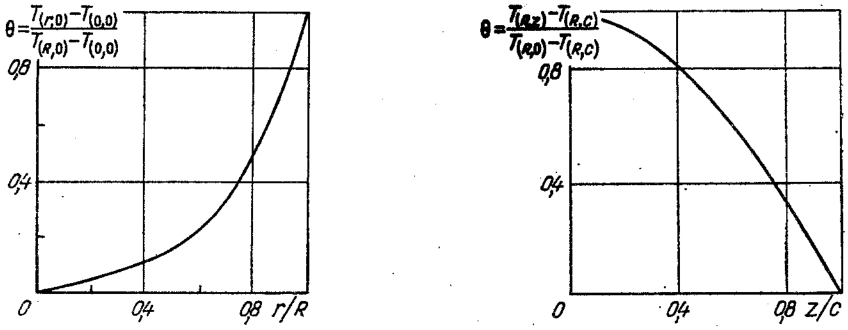

Fig. 2. Temperature distribution along the radius (r) and the generator (z) of the cylindrical sensor.

$$\frac{\partial}{\partial z} T(r,\ l) = 0. \tag{6}$$

Using Eqs. (5) and (6) we carry out a finite integral cosine Fourier transformation with respect to z [4]:

$$r \frac{d^2}{dr^2} T(r,\ p) + \frac{d}{dr} T(r,\ p) - r p^2 T(r,\ p) = 0. \tag{7}$$

The boundary conditions are:

$$\alpha \left[T(R,\ p) - T_m \frac{l \sin \frac{p\pi c}{l}}{p\pi} \right] + \lambda \frac{dT(R,\ p)}{dr} = \frac{g l \sin \frac{p\pi c}{l}}{p\pi} \tag{8}$$

$$T(0,\ p) \neq \infty. \tag{9}$$

Equation (7) is a modified Bessel equation [2], and, taking account of Eqs. (8) and (9), its solution has the form:

$$T(r,\ p) = \frac{(g + \alpha T_m)\, l \sin \frac{p\pi c}{l}}{\pi p \left[\frac{I_0(pR)}{I_0(pr)} + \lambda p \frac{I_1(pR)}{I_0(pR)} \right]}. \tag{10}$$

Applying an inverse Fourier transformation and introducing Bi = αR/λ, we obtain

$$T(r,\ z) = \frac{c(g + \alpha T_m)}{\alpha l} + 2 \sum_{p=1}^{\infty} \frac{(g + \alpha T_m) \sin \frac{p\pi c}{l}}{\alpha \pi p \left[\frac{I_0(pR)}{I_0(pr)} + \frac{pR}{\mathrm{Bi}} \frac{I_1(pR)}{I_0(pR)} \right]}. \tag{11}$$

As can be seen from the graphs (Fig. 2) constructed from Eq. (11), the operating cylindrical sensor has a temperature gradient along the radius and the length, leading to heat transfer from the surface heat source to the cylindrical substrate.

Allowing for Eq. (11) the average temperature of the film element is:

$$T = \frac{1}{2c} \int_0^{2c} T(R,\ z)\, dz = \frac{c(g + \alpha T_m)}{\alpha l} + \frac{2g}{\alpha c} \sum_{p=1}^{\infty} \frac{l \sin^2 \frac{p\pi c}{l}}{\pi^2 p^2 \left[1 + \frac{pR}{\mathrm{Bi}} \frac{I_1(pR)}{I_0(pR)} \right]}. \tag{12}$$

Then the total heat transfer from the film element to the stream and the substrate can be determined from the formula [5]

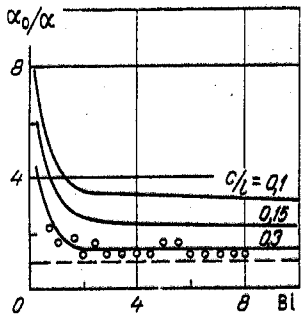

Fig. 3. Relation between the total heat-transfer coefficient α_0 and the convective coefficient α for the cylindrical sensor.

$$\alpha_0 = \frac{g}{T}. \tag{13}$$

From Eqs. (12) and (13) for $T_m = 0$ we obtain:

$$\frac{\alpha_0}{\alpha} = \left\{ \frac{c}{l} + 2\frac{l}{c} \sum_{p=1}^{\infty} \frac{\sin^2 \frac{p\pi c}{l}}{\pi^2 p^2 \left[1 + \frac{pR}{\mathrm{Bi}} \frac{I_1(pR)}{I_0(pR)} \right]} \right\}^{-1} \tag{14}$$

It can be seen from the graph (Fig. 3), constructed according to Eq. (14), that the substrate of the cylindrical sensor ceases to have a substantial influence on the sensor operation for Bi > 2 ($c/l > 0.3$). This boundary is less than that for the influence of the substrate on a film sensor made in the form of dielectric plate with a metal film on one surface [5]. An experimental check of Eq. (14) was made by comparing the calibration characteristics of a cylindrical film sensor ($c/l = 0.5$) in water and air. A certain amount of divergence between the experimental data and the theoretical curves (Fig. 3) can be explained by heat loss through the film leads.

Thus, to reduce the influence of the substrate on the measuring characteristics of the sensor film of a thermal anemometer one should increase the ratio c/l, which makes the heat loss from the substrate minimal.

NOTATION

$T(r, z, \varphi)$, sensor substrate temperature, °K; r, z, φ, coordinate of a point in a cylindrical system; a, thermal diffusivity, m/sec; α, coefficient of convective heat transfer, W/m · K; λ, thermal conductivity of the substrate, W/m · K; g, amount of heat released per unit time per unit area of film carrying current, J/sec · m; T_m, temperature of the medium washing the cylinder, °K; p, Fourier transform parameter; $I_0(Rp)$, $I_0(rp)$, $I_1(Rp)$, $I_1(rp)$, modified Bessel functions.

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